

Chiral Vacuum Alignment in Dense QCD

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Abstract

We discuss an interesting possibility of nontrivial, quark-mass induced chiral vacuum alignment in color-flavor locking phase of cold, dense QCD. With the simplifying assumption that the gaps for particles are identical to those of antiparticles and the light quark masses are given by $m_u = m_d$ and $m_s/m_d = 15$, we find the true chiral vacuum can align only in one of the discrete number of directions in the continuum of chiral vacua. The alignment depends on the diquark condensates, and the vacuum transition from one direction to another caused by the evolution of the condensates can be a first order phase transition with vanishing or nonvanishing latent heat depending on the directions involved.

The chiral symmetry $SU_L(3) \times SU_R(3)$ of QCD with three massless quark flavors is spontaneously broken to $SU_{L+R}(3)$ at low energies. Any point in the continuum of the chiral vacua, the coset space $[SU_L(3) \times SU_R(3)]/SU_{L+R}(3)$, can be chosen as a vacuum, since any two vacua are equivalent as far as physics is concerned. However when quarks receive small (current) masses the chiral symmetry is then explicitly broken, and the quark mass term picks up, following Dashen's procedure [1], a unique vacuum out of the continuum of vacua. Dashen has shown three decades ago that the correct chiral vacuum is the one that minimizes the potential $V(\Sigma) = -\mathcal{L}_m(\Sigma) \propto -\text{Re}[\text{Tr}(m\Sigma)]$, where

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Σ denotes the pseudo Goldstone boson fields and $\mathcal{L}_m(\Sigma)$ is the quark-mass induced meson mass term in the chiral Lagrangian.

When the vacuum is nontrivial, that is, $\langle \Sigma \rangle \neq \mathbf{1}$, some interesting phenomena could arise. For example, if the quark mass matrix were given as $m \propto \text{Diag}(-1, -1, -\delta)$ with $\delta > 1/2$, then the vacuum $\langle \Sigma \rangle$ would have an imaginary part in its matrix elements, and this could give rise to a spontaneous CP violation, enabling, for example, η decay into two pions. Of course, with the quark mass matrix realized in nature, the vacuum is trivial and there is no such CP violation.

Recently, cold, dense quark system with large baryon chemical potential μ has received considerable interest [2]. As well known, the Fermi surface of such a dense system is unstable against Cooper pairing when an attractive force is present. At large μ a dressed gluon exchange (in hard dense loop approximation) provides such an attractive force, and causes the system to be in color-flavor locking phase in which quarks condensate in a pattern [2]

$$\langle \chi_i^a \chi_j^b \rangle = -\langle \bar{\varphi}_i^a \bar{\varphi}_j^b \rangle \propto k_1 \delta_i^a \delta_j^b + k_2 \delta_j^a \delta_i^b, \quad (1)$$

where χ_i^a, φ_i^a , a the color index, $i = 1, \dots, 3$ the flavor index, denote two-component Weyl fermions for the left-handed quarks and the complex conjugate of right-handed quarks, respectively. Upon the diquark condensation the symmetry of dense, massless QCD, $SU_L(3) \times SU_R(3) \times U_A(1) \times U_B(1)$, is spontaneously broken to $SU_{L+R}(3)$, generating 10 Nambu-Goldstone bosons (mesons). The $U_A(1)$, which is anomalous at zero density, is a good symmetry at high density because the instanton effects are screened out at large baryon chemical potential [3, 4].

As in vacuum QCD a continuum of chiral vacua $[SU_L(3) \times SU_R(3) \times U_A(1) \times U_B(1)]/SU_{L+R}(3)$ arises upon the spontaneous symmetry breaking, and the true chiral vacuum when quark mass effect is included can be picked up by minimizing the quark mass induced potential $V(\Sigma) = -\mathcal{L}_m(\Sigma)$.

The meson mass term $\mathcal{L}_m(\Sigma)$ in color-flavor locking phase has a different form than in vacuum QCD. For small quark masses, $\mathcal{L}_m(\Sigma)$ can be expanded in powers of the quark mass matrix, and the absence of a left-right quark condensate at large μ due to

the suppression of instanton effects renders the leading term to be quadratic in quark mass [2]. The most general form for $\mathcal{L}_m(\Sigma)$ consistent with the chiral symmetry and the condensate (1) is given by [5, 6, 7, 8, 9]

$$\mathcal{L}_m(\Sigma) = A [\text{Tr}(m^t \Sigma)]^2 + B \text{Tr}[(m^t \Sigma)^2] + C \text{Tr}(m^t \Sigma) \text{Tr}(m^* \Sigma^\dagger) + h.c., \quad (2)$$

where Σ , a 3×3 unitary matrix, represents the 9 mesons. Note that the Nambu-Goldstone boson associated with the baryon number symmetry remains massless, and thus does not appear in the meson mass term.

Generally, the coefficients A, B and C depend on the chemical potential as well as the gaps for quarks and antiquarks. For simplicity, however, we shall assume here that quarks and antiquarks have identical gap parameters k_i . Then the coefficients can be determined by integrating out the quark fields in the following effective Lagrangian [5]

$$\begin{aligned} \mathcal{L} = & i\bar{\chi}_i^a \bar{\sigma}^\nu \partial_\nu \chi_i^a + \mu \bar{\chi}_i^a \bar{\sigma}^0 \chi_i^a + i\bar{\varphi}_i^a \bar{\sigma}^\nu \partial_\nu \varphi_i^a - \mu \bar{\varphi}_i^a \bar{\sigma}^0 \varphi_i^a - [m_{ij} \chi_i^a \varphi_j^a + h.c.] \\ & + [\chi_i^a (\Delta_\chi^\dagger)_{ij}^{ab} \chi_j^b - \varphi_i^a (\Delta_\varphi)_{ij}^{ab} \varphi_j^b + h.c.] + \mathcal{L}_{\text{NG}}(U, V), \end{aligned} \quad (3)$$

where

$$(\Delta_\chi^\dagger)_{ij}^{ab} = k_1 U_i^{*a} U_j^{*b} + k_2 U_j^{*a} U_i^{*b}, \quad (\Delta_\varphi)_{ij}^{ab} = k_1 V_i^a V_j^b + k_2 V_j^a V_i^b. \quad (4)$$

Here $\mathcal{L}_{\text{NG}}(U, V)$ is the usual chiral Lagrangian for the Nambu-Goldstone bosons alone [10, 11], and also supposed to contain the mass term $\mathcal{L}_m(\Sigma)$. The Lagrangian (3) may be regarded as an effective Lagrangian before color is gauged for the quarks and the 18 Nambu-Goldstone bosons U, V that arise from the spontaneous symmetry breaking of global color-chiral-axial-baryon-number symmetry. Upon gauging color it can be seen that 8 out of the 18 Nambu-Goldstone bosons are eaten by the gluons via Higgs mechanism and there remain 10 color-singlet mesons as low energy excitations.

Integrating out the quark fields, which corresponds to evaluation of quark one-loop diagrams in the background of constant U, V fields with two Dirac mass insertions, we obtain $\Sigma = UV^\dagger$ and

$$A = i \int \frac{d^4 p}{(2\pi)^4} \left\{ (k_1^2 + k_2^2) I_{8-}(p) I_{8+}(p) + k_1 (k_1 + k_2/3) [I_{8-}(p) I_{8+}(p) \right.$$

$$\begin{aligned}
& -I_{1+}(p)I_{8-}(p)] + k_1(k_1 + k_2/3)[I_{8-}(p)I_{8+}(p) - I_{1-}(p)I_{8+}(p)] \\
& + (k_1 + k_2/3)^2[I_{8-}(p)I_{8+}(p) - I_{1-}(p)I_{8+}(p) - I_{1+}(p)I_{8-}(p) \\
& + I_{1-}(p)I_{1+}(p)]\} \\
B &= i \int \frac{d^4 p}{(2\pi)^4} \{2k_1 k_2 I_{8-}(p)I_{8+}(p) + k_2(k_1 + k_2/3)[I_{8-}(p)I_{8+}(p) \\
& - I_{1+}(p)I_{8-}(p)] + k_2(k_1 + k_2/3)[I_{8-}(p)I_{8+}(p) - I_{1-}(p)I_{8+}(p)]\} \\
C &= -i \frac{1}{9} \int \frac{d^4 p}{(2\pi)^4} [-(p_0 - \mu)^2 + |\vec{p}|^2] \{I_{8-}(p)I_{8+}(p) - I_{1-}(p)I_{8+}(p) \\
& - I_{1+}(p)I_{8-}(p) + I_{1-}(p)I_{1+}(p)\}, \tag{5}
\end{aligned}$$

where

$$I_{1\mp}(p) = 1/[-p_0^2 + (|\vec{p}| \mp \mu)^2 + m_1^2], \quad I_{8\mp}(p) = 1/[-p_0^2 + (|\vec{p}| \mp \mu)^2 + m_8^2]. \tag{6}$$

Here m_1 and m_8 are the Majorana masses for the singlet and octet quarks, respectively, under the unbroken $SU_{L+R}(3)$ and are given as

$$m_1^2 = (3k_1 + k_2)^2, \quad m_8^2 = k_2^2. \tag{7}$$

Generally the diquark condensates depend on energy, but we shall ignore this fact and treat k_i as constants. Then the integration over the loop momentum can be easily performed by doing contour integration over p_0 first and then replacing $d^3 p \rightarrow 4\pi\mu^2 d|\vec{p}|$ for the integration over the spatial components. It can be easily seen that the constants are real, and at large μ , $A, B \sim \Delta^2 \ln(\mu^2/\Delta^2)$ and $C \sim \Delta^4/\mu^2 \ln(\mu^2/\Delta^2)$, where $\Delta \sim k_i$. Note that C is suppressed by a factor Δ^2/μ^2 compared to A, B .

To determine the true chiral vacuum at a given chemical potential we have to minimize the potential $V(\Sigma) = -\mathcal{L}_m(\Sigma)$, with $\mathcal{L}_m(\Sigma)$ defined by (2) and (5). To simplify, we shall now assume the up and down quark masses are identical and the quark mass matrix is given in the form

$$m = m_d \text{Diag}(1, 1, \delta), \quad \delta \equiv m_s/m_d > 0. \tag{8}$$

Note that the strange and down quark mass ratio δ can always be assumed positive, since the phase of a diagonal element of the quark mass matrix can be rotated away by

the chiral-axial transformation of the Σ field. With the quark mass matrix (8) it is easy to see that the Σ that minimizes the potential must be of the form

$$\Sigma = \text{Diag}(\alpha, \alpha, \beta) \quad \text{with} \quad |\alpha|^2 = |\beta|^2 = 1 \quad (9)$$

where α, β are complex variables.

Substituting (8) and (9) into (2) we obtain

$$V = -2m_d^2 \left\{ \text{Re}[A(2\alpha + \beta\delta)^2 + B(2\alpha^2 + \beta^2\delta^2)] + C|2\alpha + \beta\delta|^2 \right\}, \quad (10)$$

which can also be written as

$$V = -8m_d^2 \left\{ (2A + B) \cos^2 \theta + (A + B) \delta^2 / 2 \cos^2 \phi + A\delta \cos(\theta + \phi) + C\delta \cos(\theta - \phi) \right\} \quad (11)$$

with $\alpha = \exp(i\theta)$ and $\beta = \exp(i\phi)$.

We first notice that the potential is symmetric under the parity $(\alpha, \beta) \rightarrow (-\alpha, -\beta)$, and so the minima of the potential must occur in pairs. Secondly we observe that for arbitrary coefficients A, B , and C the potential is stationary when θ, ϕ satisfy $\sin(\theta \pm \phi) = 0, \sin(2\theta) = 0$, and $\sin(2\phi) = 0$, which have 8 common solutions corresponding to the following (α, β) pairs

$$(-i, -i), (-1, 1), (1, 1), (i, -i) \quad (12)$$

and their parity partners. Of course none of these pairs needs necessarily minimize the potential, but it can be shown numerically that the minima occur always on one of these solutions. This shows then that the true chiral vacuum can align only to one of these eight directions.

In ideal situation one may know the dependence of the gap parameters k_i on μ , could choose the true chiral vacuum at a given chemical potential from the above vacua, and investigate the vacuum transition from one direction to another as the chemical potential evolves. However, presently there is no reliable calculation to determine the gaps as functions of the chemical potential except when the chemical potential is extremely large ($\geq 10^8$ MeV) [12], in which case Shwinger-Dyson equation can be used as an approximate gap equation and solved [13]. Even in this case the absolute magnitudes

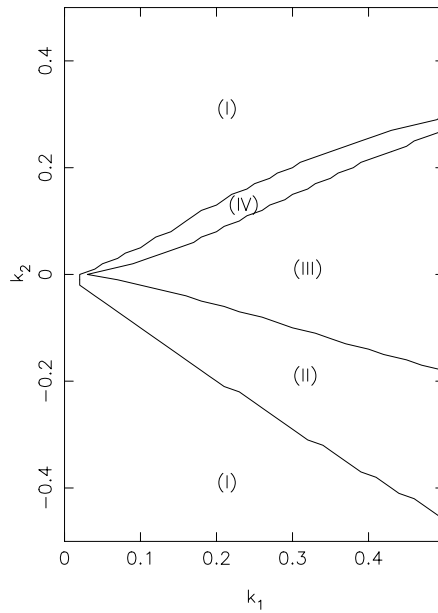


Figure 1: Gap parameter space is divided into domains according to their vacua. Each domain is associated with a unique vacuum. k_i are in the unit of $\mu = 1$.

of the gaps are yet to be determined, but it may not be so unreasonable to assume that k_i/μ are in the order of 0.1 at large chemical potential [13].

Taking into account the uncertainty in the calculation of the gaps we shall here treat k_i as free parameters and study the vacuum transitions as k_i vary. For definiteness, we shall put $\delta = 15$ and scan numerically the minima of the potential in the k_i parameter space defined by $0 \leq k_1/\mu \leq 0.5$ and $-0.5 \leq k_2/\mu \leq 0.5$. Note that k_1 can always be assumed positive, since its phase can be rotated away by the baryon number symmetry in the Lagrangian (3).

The result of the numerical scanning is shown in Fig.1. As we see, the parameter space is divided into four domains according to their vacuum directions. Note that the parity partners of the vacua associated with (12) are not included in the figure. The reason is that it can be shown numerically that in the infinite volume limit the transitions between the vacua defined by (12) and their parity partners are negligibly small, and thus only the transitions between the vacua in (12) need be considered.

Since the vacuum can align only to a discrete number of directions the transitions

between vacua will be of first order. To understand what happens at the transition, we study more carefully the potential on the boundaries between the domains. For simplicity we shall designate the domains associated with the vacua defined in (12) by (I),(II),(III), and (IV), respectively.

First, consider a transition between the domain (I) and (II). As one approaches the boundary from either side of (I) and (II) it can be seen that a potential valley opens up in (θ, φ) space along the line $\varphi = -\theta - \pi$, with the bottom of the valley connecting $(-\pi/2, -\pi/2)$ and $(-\pi, 0)$. When exactly on the boundary, we have from $V(\Sigma_I) = V(\Sigma_{II})$, with $\Sigma_{I,II}$ defined by (9) and (12),

$$(2A + B) + (A + B)\delta^2/2 - 2C\delta = 0. \quad (13)$$

Then it is trivial to see that on the bottom of the valley (i.e. along the direction $\varphi = -\theta - \pi$) the potential is constant with $V = 8m_d^2(A - C)\delta$. Thus in this transition there would be no latent heat released.

Similarly, on the boundary between (II) and (III) we have

$$A + C = 0, \quad (14)$$

and can see that a valley opens up along the line $\varphi = 0$ as one approaches the boundary. On the bottom of the valley the potential is given by

$$V = -8m_d^2 \left\{ (2A + B) \cos^2 \theta + (A + B)/2\delta^2 \right\}, \quad (15)$$

which shows a barrier between the vacua. This then indicates that there would be latent heat released at the transition.

In a similar fashion we can easily show that for the transition across the domain (III) and (IV) the potential valley opens up along the direction $\phi = -\theta$, and on its bottom the potential is constant with $V = -8m_d^2(A - C)\delta$. In this case there would be no latent heat released as in the transition between (I) and (II). For the transition between (I) and (IV) the valley opens up along $\varphi = -\pi/2$ direction and on the bottom of the valley the potential is given by

$$V = -4m_d^2(A + B)\delta^2 \cos^2 \theta, \quad (16)$$

which shows a barrier between the vacua, and consequently, nonzero latent heat at the transition.

Could a phase transition similar to those above occur when the chemical potential increases from zero to an asymptotic value? Although it is difficult to answer this question conclusively until we have the meson potential at an arbitrary μ , which would contain the instanton-induced potential that gives the η' mass as well as the traditional $\text{Tr}(m\Sigma)$ and $\mathcal{L}_m(\Sigma)$ in (2), there is an interesting observation concerning this question. It is well known that at large chemical potential the sextet components of the condensates (1) is suppressed, and thus $k_1 \approx -k_2$, which then suggests the system must be in domain (I) at large μ . The vacuum associated with domain (I) is $\langle \Sigma \rangle = \text{Diag}(-i, -i, -i)$ that has an overall factor $-i$ compared to the vacuum $\langle \Sigma \rangle = \mathbf{1}$ at zero density. Although this does not imply a phase transition, at least it does suggest that the vacuum must shift from unit matrix at zero density as the chemical potential increases.

In this letter we have studied quark-mass induced chiral vacuum alignment in cold, dense QCD. When quarks are massless any point in the continuum of chiral vacua can be chosen as a vacuum, but when nonzero quark masses are introduced, the true vacuum must be found via Dashen's procedure. We have shown that in color-flavor locking phase at large chemical potential the vacuum can align only in a discrete number of directions, and the nature of the vacuum transitions is of first order with vanishing or nonvanishing latent heat. Our conclusions were based on a few assumptions, which include a simplified quark mass matrix as well as that the gaps for particles and antiparticles are identical. However, we believe our main result that chiral vacuum can align only to a discrete number of directions is a generic feature of color-flavor locking phase, independently of the particular assumptions. We finally remark on the calculation of the meson masses. Usually in meson mass calculation it was assumed that the vacuum is at $\langle \Sigma \rangle = \mathbf{1}$ and the meson potential was expanded around this vacuum to pick up the meson spectrum. However, as we have seen, the vacuum is not necessarily at $\langle \Sigma \rangle = \mathbf{1}$ and so this is not always correct. For correct meson masses one must first find the true vacuum and then expand the potential around it.

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